Key

Math 4

Name ______________________

5-2 Difference Quotients and Rates of Change

Date________

In this Activity, you will be working towards the following learning goals:
I can compute and interpret average rates of change in functions
I can calculate and use the difference quotient for a function

VIDEOS: **"What is Calculus?"** http://www.youtube.com/watch?v=ismnD_QHKkQ
"A Brief Introduction to Calculus" http://www.youtube.com/watch?v=6gvtr_H1h90

Answer the questions based off of the videos:

1. What is calculus?

   The mathematics of change

2. What is a difference between algebra & calculus?

   Algebra: More regular, less real
   Calculus: less regular, more real → real life applications

3. Since what year has calculus been around?

4. What were the main problems Newton & Leibniz were concerned about solving?

5. "Climbing down the ladder" – what tool in calculus? What is happening to the powers?

SKIP!!

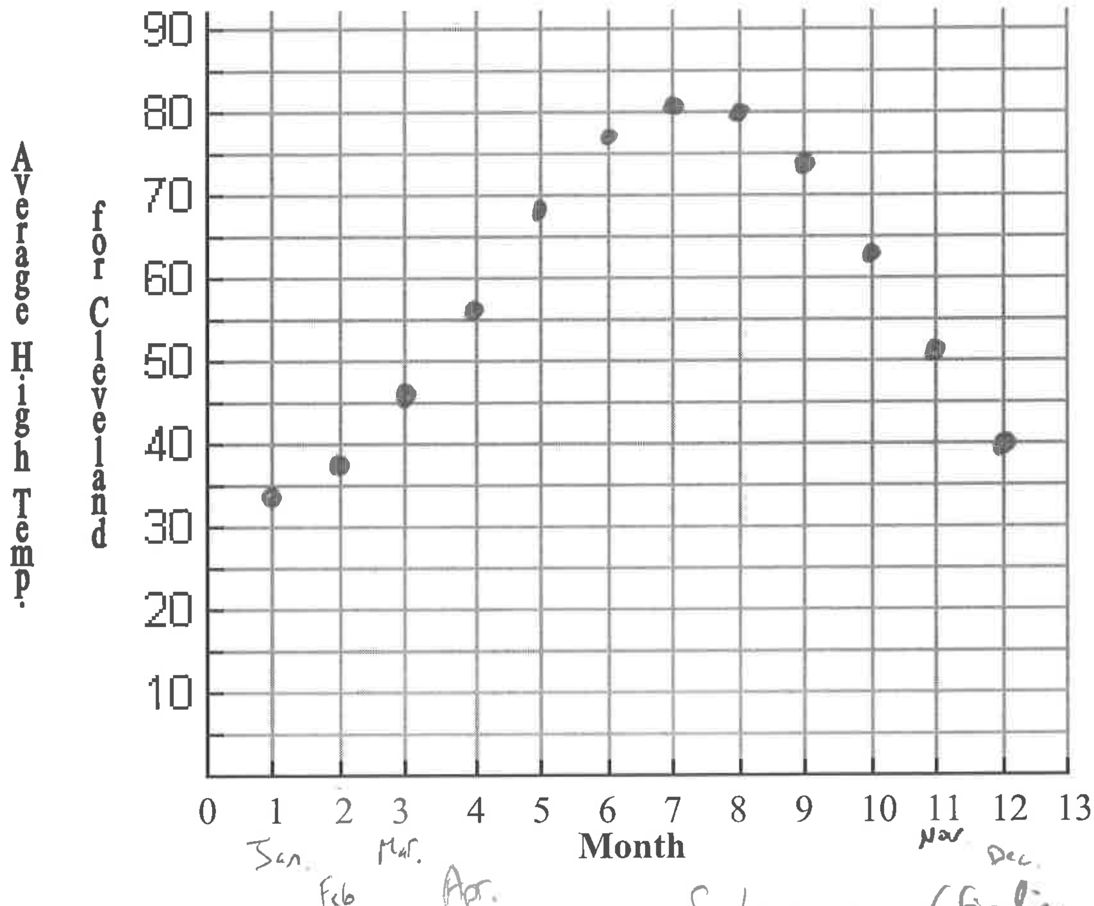
6. "Climbing up the ladder" – what tool in calculus? What is happening to the powers?

7. The two components of calculus are: ____________________ & ______________________

Please meet with your group members & discuss your answers.

II. In this portion of the lesson, you will investigate **rates of change**. Plot the following data points on the graph below. Then answer the questions.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Average High Temp. in ° F	34	37	46	56	68	77	81	80	74	63	51	40



1. To calculate *change*, what operation is used? Subtraction (finding the difference)

2. What Greek letter is used to represent change? Δ (Delta)

   In symbols, change in month x_1 to month x_2 = $X_2 - X_1$ = Δx

   In symbols, change in temp y_1 to temp y_2 = $Y_2 - Y_1$ = (or in function notation) $f(x_2) - f(x_1)$ = Δy

3. The *average rate of change* is found by dividing the changes.

   Average Rate of Change (A_{RoC}) = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ = (in function notation): $\boxed{\dfrac{f(x_2) - f(x_1)}{X_2 - X_1}}$

4. Calculate the following (use the correct units):

A_{RoC} Feb → Jul :

$$A_{RoC} = \frac{81 - 37}{7 - 2} = \frac{44}{5} = \boxed{8.8 \text{ degrees per Month}}$$

A_{RoC} Jul → Dec :

$$A_{RoC} = \frac{40 - 81}{12 - 7} = \frac{-41}{5} = \boxed{-8.2°/\text{mo.}}$$

The Geometric Definition of **Average Rate of Change:**

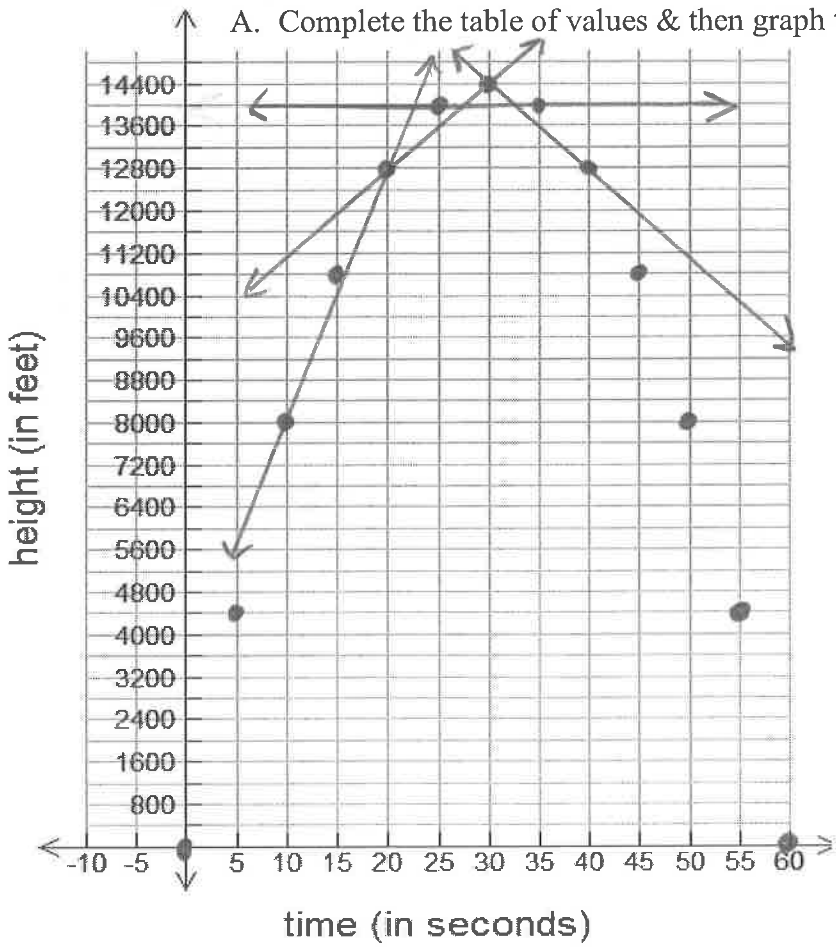
The slope of the line through $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

***If we connect the two points, we have what is called a **secant line** for the graph of the function.

$$m_{\text{secant}} = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{\Delta y}{\Delta x}$$

Example: A projectile follows along a path given by the formula $h(t) = 960t - 16t^2$.

A. Complete the table of values & then graph the path of the object.



t	$h(t)$
0	0
5	4400
10	8000
15	10800
20	12,800
25	14,000
30	14,400
35	14,000
40	12,800
45	10,800
50	8000
55	4400
60	0

B. Use the formula from above to calculate $\mathbf{A_{RoC}}$ over the following intervals (use the correct units):

1.) $10 \le t \le 20$

$= \frac{12,800-8000}{20-10} = \frac{4800}{10} = \boxed{480 \text{ ft/s}}$

2.) $20 \le t \le 30$

$= \frac{14,400-12,800}{30-20} = \frac{1600}{10} = \boxed{160 \text{ ft/s}}$

3.) $30 \le t \le 40$

$= \frac{12800-14,400}{40-30} = \frac{-1600}{10} = \boxed{-160 \text{ ft/s}}$

4.) $25 \le t \le 35$

$= \frac{14000-14000}{35-25} = \frac{0}{10} = \boxed{0 \text{ ft/s}}$

C. What do your computations in part B tell you about the projectile?

We calculated the projectile's average velocity (speed with direction) over the given intervals

D. Use a ruler to draw the secant lines going through the pairs of points. Do the direction of the lines confirm the signs of your computations in part B?

Yes! Positive slope = positive A_{RoC}

Neg. slope = Neg. A_{RoC}

0 slope = horizontal line

***__Average Velocity__ over an interval – the average rate of change of *directed* distance.

If average velocity > 0 → The projectile is going up.

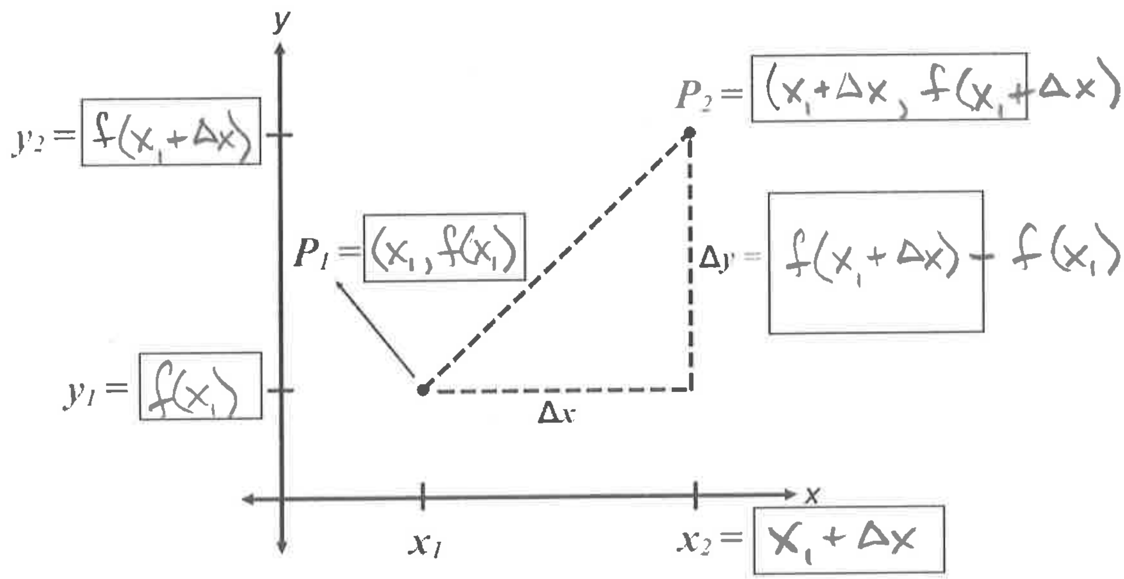
If average velocity < 0 → The projectile is going down.

If average velocity = 0 → The projectile is at the same height at both times.

__Part 3__: The concept of **ARoC** is an important one in mathematics. When many **ARoC**s have to be calculated for a particular function, *f*, it helps to have a general formula called the **Difference Quotient**. Below you will derive a general formula to help write the difference quotient for any function, *f*.

We're going to start by representing the difference quotient geometrically/graphically. Take notes and fill in the boxes within the figure below with equivalent expressions:

$$\text{Recall: } \begin{pmatrix} \text{average rate} \\ \text{of change in} \\ f \text{ from } x_1 \text{ to } x_2 \end{pmatrix} = \frac{\Delta y}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2 - x_1}$$



Now we're going to use the information from the graph above to write the **ARoC** equation without the x_2 terms. Remember, $\Delta x = x_2 - x_1$.

Write your new formula here: The **Difference Quotient** = $\boxed{\dfrac{f(x_1+\Delta x) - f(x_1)}{\Delta x}}$

$x_1 + \Delta x - x_1$

Example: Refer to the projectile example, where $h(t) = 960t - 16t^2$. Find a formula for the difference quotient given the average rate of change of h for each interval t to $t + \Delta t$.
****Note: Δt is one variable!*

$$\frac{h(t+\Delta t)-h(t)}{\Delta t} = \frac{960(t+\Delta t) - 16(t+\Delta t)^2 - (960t - 16t^2)}{\Delta t}$$

$$= \frac{960t + 960\Delta t - 16(t^2 + 2t\Delta t + \Delta t^2) - 960t + 16t^2}{\Delta t}$$

$$= \frac{\cancel{960t} + 960\Delta t \cancel{-16t^2} - 32t\Delta t - 16\Delta t^2 \cancel{-960t} \cancel{+16t^2}}{\Delta t}$$

$$= \frac{960\Delta t - 32t\Delta t - 16\Delta t^2}{\Delta t}$$

$$= \frac{\cancel{\Delta t}(960 - 32t - 16\Delta t)}{\cancel{\Delta t}}$$

$$= \boxed{960 - 32t - 16\Delta t}$$

include units!

A. Use the results from above and $t = 5$ to find the average velocity when $\Delta t = \ldots$

a.) $\Delta t = 1$ 1 second change

$= 960 - 32(5) - 16(1)$

$= 960 - 160 - 16$

$= \boxed{784 \text{ ft/s}}$

b.) $\Delta t = .5$ 0.5 second change

$= 960 - 32(5) - 16(0.5)$

$= 960 - 160 - 8$

$= \boxed{792 \text{ ft/s}}$

c.) $\Delta t = .1$ $\frac{1}{10}$ second change

$= 960 - 32(5) - 16(0.1)$

$= 960 - 160 - 1.6$

$= \boxed{798.4 \text{ ft/s}}$

d.) $\Delta t = .01$ $\frac{1}{100}$ second change

$= 960 - 32(5) - 16(.01)$

$= 960 - 160 - .16$

$= \boxed{799.84 \text{ ft/s}}$

B. What is happening to the Δt values in part A? What value does the average velocity appear to be approaching? Δt values approaching 0. Average velocity approaching 800 ft/s

Complete: $\lim_{\Delta t \to 0}\left(\frac{h(5+\Delta t)-h(5)}{\Delta t}\right) =$ 800

Let $h(t) = 12t^2 - 8t$. Find $\dfrac{h(t+\Delta t) - h(t)}{\Delta t}$

$$= \frac{12(t+\Delta t)^2 - 8(t+\Delta t) - (12t^2 - 8t)}{\Delta t}$$

$$= \frac{12(t^2 + 2t\Delta t + \Delta t^2) - 8t - 8\Delta t - 12t^2 + 8t}{\Delta t}$$

$$= \frac{\cancel{12t^2} + 24t\Delta t + 12\Delta t^2 - \cancel{8t} - 8\Delta t \cancel{- 12t^2} \cancel{+ 8t}}{\Delta t}$$

$$= \frac{24t\Delta t + 12\Delta t^2 - 8\Delta t}{\Delta t}$$

$$= \boxed{24t + 12\Delta t - 8}$$

Let $k(t) = 2t^2 + 5t$. Find $\dfrac{k(t+\Delta t) - k(t)}{\Delta t}$

$$= \frac{2(t+\Delta t)^2 + 5(t+\Delta t) - (2t^2 + 5t)}{\Delta t}$$

$$= \frac{2(t^2 + 2t\Delta t + \Delta t^2) + 5t + 5\Delta t - 2t^2 - 5t}{\Delta t}$$

$$= \frac{\cancel{2t^2} + 4t\Delta t + 2\Delta t^2 \cancel{+ 5t} + 5\Delta t \cancel{- 2t^2} \cancel{- 5t}}{\Delta t}$$

$$= \frac{4t\Delta t + 2\Delta t^2 + 5\Delta t}{\Delta t}$$

$$= \boxed{5 + 4t + 2\Delta t}$$